



**B.ED.- I (SEMESTER-I) PAPER IV & V (OPTION IX)
TEACHING OF MATHEMATICS**

Pedagogy of a school subject

Section: B

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Punjabi University, Patiala**

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LESSON NO :

SECTION - B

- 2.1 : Aims and objectives of teaching mathematics at elementary and secondary schools.
- 2.2 : Bloom's Taxonomy of educational objectives and writing objectives in behavioural terms
- 2.3 : Pedagogical analysis: An overview
- 2.4 : Pedagogical Analysis of certain unit in Mathematics-I
- 2.5 : Pedagogical Analysis of certain unit in Mathematics-II

Aims and objectives of Teaching Mathematics at Elementary and Secondary Schools

Structure of the lesson

2.1.1 Objectives

2.1.2 Introduction

2.1.3 Difference between Aims and Objectives

2.1.4 Aims of Teaching Mathematics at Elementary and Secondary Schools

2.1.5 Objectives of Teaching Mathematics at Elementary and Secondary Schools

2.1.5.1 Knowledge and Understanding objectives

2.1.5.2 Skill Objectives

2.1.5.3 Application Objectives

2.1.5.4 Attitude Objectives

2.1.5.5 Appreciation Objectives

2.1.6 Summary

2.1.7 Suggested Questions

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2.1.1 Objectives

After going through this lesson learners will be able to:

- i. Define aims and objectives
- ii. Differentiate between aims and objectives

- iii. Explain the aims of teaching mathematics at elementary and secondary schools
- iv. Explain the objectives of teaching mathematics at elementary and secondary schools
- v. Differentiate between different objectives of teaching mathematics in elementary and secondary schools

2.1.2 Introduction

In school curriculum every subjects have its own place and importance. Until we have clear aims of teaching a subject, we would not be able to proceed on the right path. As we know education is a process based on aims. So no teaching process can be complete without aims. Aimlessness makes the work tedious and outcomes in the wastage of time, energy and other resources. So, we must have some definite aims of teaching a subject before starting its actual teaching. Aims support in the understanding of values or drawing of advantages whiles the knowledge of the values of subject help in setting the aims to get all the necessary advantages. This makes us to conclude that the knowledge of the values of teaching mathematics (discussed in Unit I) may help us a lot in setting the aims of teaching mathematics in our school.

2.1.3 Aims and Objectives

Aims and objectives for student learning serve as a means of clarifying our determined as teachers. They should assist in selection of teaching method, strategies, design and evaluation of the effectiveness of a teaching program.

Aims and objectives both consist of two essential parts; an action verb and subject content. They are written from the perception of the learners; they are what the learners can do upon end of the learning.

Difference between Aims and Objectives

Aims	Objectives
It needs a long term planning	It needs a short term planning
It is a broader concept	It is a narrower concept
It is not clear and uncertain	It is clear and certain
It takes more time to achieve	It takes less time to achieve

It is not possible to measure it	It is possible to measure it
It includes the objectives	It is a part of aim
It is difficult to achieve	It can be achieve easily
Its origin is sociology, philosophy	Its origin is psychology
Its relation to future and education	It is included in education and teaching

No subject can be taught without having aims and objectives of teaching of that subject clearly before the eyes. They provide proper direction to the teacher and the students.

2.1.4 Aims of Teaching Mathematics at Elementary and Secondary Schools

Thurber and Collette have suggested following criteria for the selection of aims and objectives:

1. Usefulness:

The desired learning should have value in the lives of the pupils.

2. Timelines:

Learning should be concerned with material familiar at the present time not with absolute devices and ideas.

3. Fitness:

The learning should fit into a sequence leading towards broad objectives.

4. Appropriateness:

The learning called for should be appropriate for the maturity and background of the pupils.

5. Practicability:

Experiences needed for the development of the learning should be possible.

Now we can prepare long lists of aims of teaching mathematics at entire school stage. The teaching of mathematics helps the students in acquiring essential mathematical knowledge, interest, skills and attitude for the following purposes.

- i) Utilitarian Aim
- ii) Intellectual Aim
- iii) Cultural Aim
- iv) Social Aim
- v) Moral Aim
- vi) Aesthetical Aim
- vii) Vocational Aim
- viii) International Aim
- ix) Inter- disciplinary Aim
- x) Self-learning Aim

2.1.5 Objectives of Teaching Mathematics at Elementary and Secondary Schools

According to NCERT's evaluation and examination issue: - An objective is a point of end in view of something towards which action is directed, a planned change sought through any activity what we set out to do.

The objectives are to be properly described and expressed in terms of the expected behavioral changes or the learning outcomes. It should be clearly indicated what the student is expected to achieve through the learning of a unit. These objectives should be classified into the important sub-categories.

Following are the functions of formulation of objectives:

- Guides for design instruction
- Guides for evaluation/test design (e.g. written tests, etc)
- Guides the learner relative to learning focus
- Guide for the learner relative to self assessment
- Enhances possibility to create learning materials.
- Makes teaching well directed and organized.
- Communicates to colleagues what you are teaching thus enhances collaboration and teamwork.
- Helps to facilitate situations in which you want your students to demonstrate competency

- Forces to think carefully about what is important
- Helps to avoid unnecessary repetitions in teaching
- Provides visibility and accountability of decisions.
- Provides models for the creation of objectives by students
- Helps students make decisions regarding prioritizing
- Provides feedback to learners as objectives are accomplished.

The objectives of teaching mathematics at the entire school stage may be classified as under:

- i) Knowledge and understanding objectives
- ii) Skill Objectives
- iii) Application Objectives
- iv) Attitude Objectives
- v) Appreciation Objectives

2.1.1 Knowledge and Understanding objectives:

Through knowledge and understanding students develop mathematical reasoning to make deductions and solve problems. Through mathematics a student acquire the knowledge and understanding of:

- Know the concepts of different branches of mathematics like algebra, geometry, trigonometry, statistics and probability
- Learns language of mathematics like mathematical symbols, formulae, figures diagram etc.
- Learns the fundamental mathematical ideas, rules and relationship
- Learns historical development , contribution of mathematicians
- Understand the inter-relationship among different topics of mathematics

2.1.2 Skill objectives

Through mathematics a student acquire the skill of:

- Learns to perform calculations
- Develops the ability to organise and interpret the given data

- Develops speed and accuracy in solving problems
- Develops the skill to draw figures and diagram
- Develops the ability to use mathematical apparatus skilfully
- Develops the ability to use mathematical tables
- Develops the ability to guess, check and verify results
- Develops skill in measuring, weighing and surveying
- Learns to express thoughts clearly and accurately

2.1.3 Application objectives

Through mathematics a student apply the above knowledge and skill in the following way:

- Apply the knowledge of mathematical concepts in day to day life
- Develops the ability to analyse and generalize the facts
- Apply the knowledge of mathematical concepts in other subjects
- Apply mathematics in his future vocational life

2.1.4 Attitude objectives

Mathematics helps in the development of attitude in the following way:

- Develops the attitude of systematically following a assignment to completion
- Develops the habit of logical reasoning
- Develops the habit of verification
- Develops the habit of critical and independent thinking
- Develops power concentration
- Develops personal qualities like regularity, honesty, neatness and truthfulness
- Develops proper self-confidence for solving the mathematical problems

2.1.5 Appreciation objectives

Through mathematics a student can learn to appreciate

- the contribution of mathematician in the development of subjects
- Role of mathematics in day to day life
- Role of mathematics in other subjects

- Role of mathematical language , graphs and tables in giving exactness and accuracy to any problem
- The cultural value of mathematics
- The value of mathematics as leisure time activity
- The vocational value of the subject mathematics
- The aesthetic value of mathematics by observing different mathematical facts

2.1.6 Summary

We have discussed the aims and objectives of teaching mathematics in general. An objective is the end point towards which each and every activity is directed in the educational process. Aims are general and long term goals and can be regarded as an expression of strategy. The teacher should carefully choose the objectives regarding a particular topic.

2.1.7 Suggested Questions

Q1 Describe the aims of teaching mathematics at the entire school stage.

Q2. What are the difference between aims and objectives?

Q3 Discuss the various objectives of teaching mathematics at the entire school stage.

Q4 Select any topic and formulate the objectives of teaching that topic in mathematics.

Q5 What is the utility of knowledge and understanding objectives to a mathematics teacher?

Q6 What is the utility of skill objectives to a mathematics teacher?

Q7 What is the utility of application objectives to a mathematics teacher?

2.1.8 Suggested reading and web resources

Adams. G.S (1964); Measurement and evaluation in education, psychology and guidance, New York: Halt. Rinehart and Winston.

Bloom,B.S., Hasting, J.T. and Madaus,G.F (1971); Handbook on formative and

summative evaluation of students learning, NewYork: Mc Graw Hill.

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Lesson No. 2.2

Bloom's Taxonomy of educational objectives and writing objectives in behavioural terms

Structure of the lesson

2.2.1 Objectives

2.2.2 Introduction (Bloom's Taxonomy)

2.2.3 Cognitive Domain

2.2.4 Affective Domain

2.2.5 Psychomotor Domain

2.2.6 Writing instructional objectives in behavioural terms

2.2.7 Summary

2.2.8 Suggested Questions

2.2.9 Suggested reading and web resources

2.2.1 Objectives

After going through this lesson learners will be able to:

- vi. Define taxonomy
- vii. Define taxonomy of instructional objectives
- viii. Differentiate between three domain of behaviour
- ix. Explain cognitive domain
- x. Explain affective domain
- xi. Explain psychomotor domain
- xii. Write instructional objectives in behavioural terms

2.2.2 Introduction

Instructional objectives: Instructional objectives in relation to the teaching of mathematics may be defined as a group of statements formulated by the teacher for describing what the pupils are expected to do or will be able to do once the process of classroom instruction is over.

Taxonomy of Instructional objectives: "Taxonomy" means 'classification', so the taxonomy of learning or educational objectives is an attempt to classify forms and levels of learning. Taxonomy of cognitive objectives developed by American academic and educational expert Dr. Benjamin Samuel Bloom with collaborators Max Englehart, Edward Furst, Walter Hill, and David Krathwohl (1956) is commonly referred as **Bloom's taxonomy**. It identifies three domains. These three domains are usually presented as hierarchies involving different levels that reflect varying degrees of complexity. The domains typically addressed by educational objectives involve cognitive, affective, or psychomotor abilities or characteristics.

1. **Cognitive domain** (intellectual capability, i.e., **knowledge**, or '**think**')
2. **Affective domain** (feelings, emotions and behavior, i.e., **attitude**, or '**feel**')
3. **Psychomotor domain** (manual and physical skills, i.e., **skills**, or '**do**')

2.2.3 Cognitive domain

Cognitive domain emphasizes on intellectual or learning outcomes. The most-used of the domains, refers to knowledge structures. This domain is further divided into six categories or levels.

1. **Knowledge:** The simplest and lowest level of the taxonomy is knowledge. It is defined as the remembering (recalling) of previously learned information. Educational objectives at the knowledge level involve learning, memorizing and also recalling a wide range of material, from specific facts to complete theories, terms, names, dates and so forth but all that is required is the bringing to mind of the appropriate information.
2. **Comprehension:** It is defined as the ability to grasp or construct meaning from the material. Educational objectives at the comprehension level require understanding, not simply rote memorization. At this level,

learning outcomes goes one step beyond the simple remembering of material, and represents the lowest level of understanding.

3. **Application:** It refers to the ability to use learned material in new and concrete situations. Educational objectives at this level involve the use of general rules, principles, or abstract concepts to solve a problem not previously encountered. Learning outcomes requires a higher level of understanding than that under comprehension.
4. **Analysis:** Analysis refers to the ability to reduce or break down material into its component parts so that its organizational structure may be understood. Educational objectives at this level require the student to break down a complex concept into its basic parts or elements in a manner that illustrates the relationship of parts to a whole. Learning outcomes here represent a higher intellectual level than comprehension and application because they require an understanding of both the content and the structural form of the material.
5. **Synthesis:** It refers to the ability to put parts together to form a new whole. Educational objectives at this level require the student to blend existing elements in such a way that they form new structures and patterns. Learning outcomes in this area stress creative behaviors, with major emphasis on the formulation of new concepts.
6. **Evaluation:** It refers to the ability of judging the value of material based on personal values/opinions, resulting in an end product, with a given purpose, without real right or wrong answers. Educational objectives at the evaluation level require the student to make evaluative judgments regarding the quality, value or worth of something for a stated purpose. Learning outcomes in this area are highest in the cognitive hierarchy because they contain elements of all the other categories, plus conscious value judgments based on clearly defined criteria.

BLOOM'S TAXONOMY OF EDUCATIONAL OBJECTIVES

Level	Description	Keywords/Action Verbs
Knowledge	Exhibits previously learned material by recalling facts, terms, basic concepts and answers.	Who, what, why, when, omit, where, which, choose, find, how, define, label, show, spell, list, match, name, relate, tell, recall, select.

Comprehension	Demonstrating understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions and stating main ideas.	Compare, contrast, demonstrate, interpret, explain, extend, illustrate, infer, outline, relate, rephrase, translate, summarize, show, classify.
Application	Solving problems by applying acquired knowledge, facts, techniques and rules in a different way	Apply, build, choose, construct, develop, interview, make use of, organize, experiment with, plan, select, solve, utilize, model, identify.
Analysis	Examining and breaking information into parts by identifying motives or causes; making inferences and finding evidence to support generalizations.	analyze, categorize, classify, compare, contrast, discover, dissect, divide, examine, inspect, simplify, survey, take part in, test for
Synthesis	Compiling information together in a different way by combining elements in a new pattern or proposing alternative solutions.	build, choose, combine, compile, compose, construct, create, design, develop, estimate, formulate, imagine, invent, make up,
Evaluation	Presenting and defending opinions by making judgments about information, validity of ideas or quality of work based on a set of criteria.	award, choose, conclude, criticize, decide, defend, determine, dispute, evaluate, judge, justify, measure, compare, mark, rate, recommend

2.2.4 Affective Domain

The taxonomy of affective objectives was developed by Krathwohl, Bloom and Masia (1964). The affective domain involves characteristics such as values, attitudes, interests, and behavioral actions. Learning is demonstrated by behaviors indicating attitudes of awareness, interest, attention, concern, and responsibility, ability to listen and respond in interactions with others.

Educational objectives involve the attitude and actions of students in relation to a school subject.

1. **Receiving:** It refers to the student's willingness to attend to particular phenomena of stimuli (classroom activities, textbook, music, etc.). Learning outcomes in this area range from the simple awareness that a thing exists to selective attention on the part of the learner. Receiving represents the lowest level of learning outcomes in the affective domain. From a teachers' point, it is concerned with getting, holding and directing the student's attention.
2. **Responding:** Responding refers to active participation on the part of the student. At this level he or she not only attends to a particular phenomenon but also reacts to it in some way. Learning outcomes in this area may emphasize acquiescence in responding (reads assigned material), willingness to respond (voluntarily reads beyond assignment), or satisfaction in responding (reads for pleasure or enjoyment). The higher levels of this category include those instructional objectives that are commonly classified under "interest"; that is, those that stress the seeking out and enjoyment of particular activities.
3. **Valuing:** It is concerned with the worth or value a student attaches to a particular object, phenomenon, or behavior. This ranges in degree from the simpler acceptance of a value (desires to improve group skills) to the more complex level of commitment (assumes responsibility for the effective functioning of the group). Valuing is based on the internalization of a set of specified values, but clues to these values are expressed in the student's overt behavior. Learning outcomes in this area are concerned with behavior that is consistent and stable enough to make the value clearly identifiable. Instructional objectives that are commonly classified under "attitudes" and "appreciation" would fall into this category.
4. **Organization:** It is concerned with bringing together different values, resolving conflicts between them, and beginning the building of an internally consistent value system. Thus the emphasis is on comparing, relating, and synthesizing values. Learning outcomes may be concerned with the conceptualization of a value (recognizes the responsibility of each individual for

improving human relations) or with the organization of a value system (develops a vocational plan that satisfies his or her need for both economic security and social service). Instructional objectives relating to the 'development of a philosophy of life' would fall into this category.

5. **Characterization by a value or value set/complex:** At this level of the affective domain, the individual has a value system that has controlled his or her behavior for a sufficiently long time for him or her to develop a characteristic "life-style." Thus the behavior is pervasive, consistent, and predictable. Learning outcomes at this level cover a broad range of activities, but the major emphasis is on the fact that the behavior is typical or characteristic of the student. Instructional objectives that are concerned with the student's general patterns of adjustment (personal, social, emotional) would be appropriate here.

KARTHWOHL'S TAXONOMY OF EDUCATIONAL OBJECTIVES

Levels	Description	Keywords
Receiving	Aware and attending to what surrounds the learners and their willingness to take heed of the stimuli	Asks, chooses, describes, follows, gives, holds, identifies, locates, names, points to, selects, sits erect, replies, uses.
Responding	Respond and interested by participating and asking some questions.	Answers, assists, complies, conforms, discusses, greets, helps, labels, performs, practices, presents, reads, recites, reports, tells, writes.
Valuing	Accept and believe by debating over the issue or making a personal stand on certain value systems.	Completes, describes, differentiates, explains, follows, forms, initiates, invites, share, studies, works.
Organization	Involve actively and commit by organizing activities like meetings, working committees, support groups related to value systems.	Adhere, alters, arranges, combines, compares, generalizes, identifies, relates, synthesizes, organizes, integrate.
Characterization	Integrating into the learners'	Acts, discriminates, displays,,

	personality to become part and parcel of their whole value system and character.	influences, listens, modifies, performs, practices, proposes, qualifies, revises, serves, solves, uses, verifies.
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2.2.5 Psychomotor Domain

The psychomotor domain concerns things students might physically do. Anita J. Harrow's (1972) taxonomy for the psychomotor domain is organized according to the degree of coordination including involuntary responses as well as learned capabilities. Simple reflexes begin at the lowest level of the taxonomy, while complex neuromuscular coordination makes up the highest levels (Seels & Glasgow, 1990).

1. **Reflex movements:** It refers to actions elicited without learning in response to some stimuli. Examples include: flexion, extension, stretch, postural adjustments.
2. **Basic fundamental movement:** It refers to inherent movement patterns which are formed by combining of reflex movements and are the basis for complex skilled movements. Examples are: walking, running, pushing, twisting, gripping, grasping, manipulating.
3. **Perceptual:** It refers to interpretation of various stimuli that enable one to make adjustments to the environment. Visual, auditory, kinesthetic, or tactile discrimination. Suggests cognitive as well as psychomotor behavior. Examples include: coordinated movements such as jumping rope, punting, or catching.
4. **Physical activities:** It requires endurance, strength, vigor, and agility which produce a sound, efficiently functioning body. Examples are: all activities which require a) strenuous effort for long periods of time; b) muscular exertion; c) a quick, wide range of motion at the hip joints; and d) quick, precise movements.
5. **Skilled movements:** The results of the acquisition of a degree of efficiency when performing a complex task. e.g., all skilled activities obvious in sports, recreation, and dance.
6. **Non-discursive communication:** It is communication through bodily movements ranging from facial expressions through sophisticated choreographic or expressive dance. For example, body postures, gestures, and facial expressions efficiently executed in skilled dance movement and choreographic.

2.2.6 Writing instructional objectives in behavioural terms

An instructional objective is a clear and unambiguous description of educational expectations for students. When written in behavioral terms, an objective will include three components:

- I. Student behavior
- II. Condition of performance
- III. Performance criteria

Thus it may be defined as a group of statements formulated by the teacher in behavioral term for describing what the pupils are expected to do or will be able to do once the progress of classroom instruction is over.

METHODS OF WRITING INSTRUCTIONAL OBJECTIVES IN BEHAVIORAL TERMS

- Robert Mager's Approach
- Robert Miler's Approach
- R.C.E.M. Approach

R.C.E.M. Approach

The four categories of objectives (knowledge, understanding, application, creativity), have been divided into 17 mental processes or abilities. These processes or abilities are used for the necessary outline of the 17 frames or statements as follows:

1. Knowledge objectives

1.1 The learner will be able to recognize...

1.2 The learner will be able to recall...

2. Understanding objectives

2.1 The learner will be able to see relationship between...and...

2.2 The learner will be able to cite example of...

2.3 The learner will be able to classify...

2.4 The learner will be able to verify...

The learner will be able to generalize...

2.5 The learner will be able to interpret...

3. Application objectives

3.1 The learner will be able to reason out...

3.2 The learner will be able to formulate hypothesis for....

3.3 The learner will be able to establish hypothesis for...

3.4 The learner will be able to infer about...

3.5 The learner will be able to predict about...

4. Creativity objectives

4.1 The learner will be able to analyze...

4.2 The learner will be able to synthesize...

4.3 The learner will be able to evaluate...

Things to be kept in mind while writing objectives in R.C.E.M. Approach

1. The entry behavior of the learner should be kept in mind.
2. Think again the element of content or topic to be given to the learner.
3. Think again the teaching objective(s)
4. In view of the entry behavior, element of content and the particular objective, try to select appropriate mental process for writing the objective in question.
5. Make use of the 17 frames of the R.C.E.M. approach and fill in the blanks in view of the entry behavior of the learner and learning experiences given to him.

2.2.7 Summary

The discussion held so far may help the teachers in the task of formulating the desired instructional objectives related to the topics or sub units of their daily lessons. The question, however, arises in actual practice and

conditions available for teachers training what type of behavioral changes part really expected in the behavior of the students through the teaching of the lessons in a particular subject. These changes falling in different domains of their behavior may generally be summarized as follows:

1. Students may acquire the knowledge and understanding of the facts, principles and ideas related to the topics and units of different branches of a particular subject of the school curriculum.
2. They may learn about the various skills related to the process and products of the subject like drawing skill, surveying skill, computational skill etc.
3. They may be able to apply the knowledge, understanding and skills related to the Subject in their daily life.
4. They may develop proper positive attitude for the learning.
5. They may develop proper interest and appreciation for the facts related to the study of that subject.

2.2.8 Suggested Questions

Q1 What do you mean by Taxonomy?

Q2 Define instructional objectives.

Q3 What do you understand by taxonomy of instructional objectives? Explain the Bloom's Taxonomy of instructional objectives related to the cognitive domain of the behaviour.

Q4 Explain in detail the Taxonomy given by Bloom and his associate for the instructional objective in affective domain of the behaviour.

Q5 Name the three main approaches employed for writing objectives in behavioural terms.

Q6 What is the RCEM approach of writing objectives in behavioural terms?

2.2.9 Suggested reading and web resources

Adams. G.S (1964); Measurement and evaluation in education, psychology and guidance, New York: Halt. Rinehart and Winston.

- Bloom, B.S., Hasting, J.T. and Madaus, G.F (1971); Handbook on formative and summative evaluation of students learning, New York: Mc Graw Hill.
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Lesson No. 2.3

Pedagogical Analysis: An Overview**Structure of the Lesson**

- 2.3.1 Objectives
- 2.3.2 Introduction
- 2.3.3 Need for pedagogical analysis
- 2.3.4 Procedure for pedagogical analysis
 - 2.3.4.1 Identification of Concepts in the unit
 - 2.3.4.2 Objectives Formulation
 - 2.3.4.3 Choosing Methods, Materials and Learning Experiences
 - 2.3.4.4 Evaluation Procedure, Tools and Devices
- 2.3.5 Summary
- 2.3.6 Suggested Questions
- 2.3.7 Suggested Readings and Web Resources

2.3.1 Objectives

After going through this lesson learners will be able to:

- i. recall the components of pedagogical analysis of a unit in Mathematics.
- ii. know the meaning of content analysis of a unit.
- iii. describe the process of content analysis of a unit.
- iv. know the meaning of objectives.
- v. describe the functions of objective formulation.
- vi. recall the meaning of terms like experience learning, materials etc.
- vii. select and apply different methods of teaching.
- viii. know the meaning of evaluation.
- ix. select and apply different techniques of evaluation in mathematics.
- x. develop meaningful questions for the purpose of evaluation.

2.3.2 Introduction:

Pedagogical analysis involves two words pedagogy and analysis. Pedagogy means that the art and the science of teaching or learner focused education for people of all ages. Pedagogy is also referred to as the correct use of teaching strategies. Analysis is the process of breaking a complex topic or substance into smaller parts to gain a better understanding of it or separation of the whole into its constituent parts in order to study the parts and their relations.

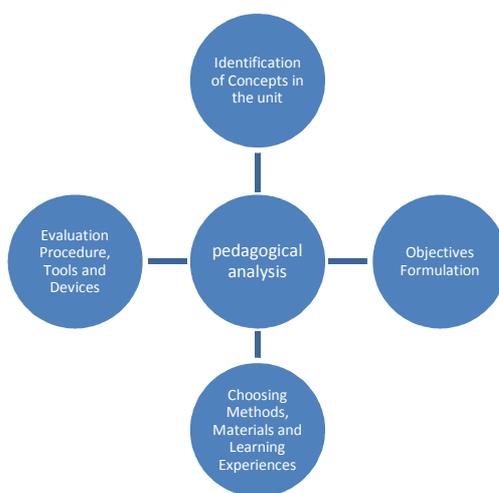
The analysis of contents of a unit of Mathematics is called the pedagogical analysis of a unit of Mathematics or unit analysis. The present lesson deals with need of the pedagogical analysis and five steps in the pedagogical analysis of a mathematical unit of study. You may call it unit analysis. Thus analysis

helps you to identify major and minor concepts to be covered in the selected unit. The objectives can be written in behavioral terms. Learning experiences can be designed. Method and materials can be selected for teaching and evaluation procedures can be decided.

2.3.3 Need for pedagogical analysis

The teacher decides instructional objectives, equipment and strategies with every aspect of learning conditions to be created. Pedagogical analysis is appropriate for objectives and strategies in various instructional situations and assesses the levels of actual learning. Pedagogical analysis offers vast prospective for improving the delivery of information in all form of education. It involves various logical steps to arrive at logical conclusion. It also helps the learners to understand concepts, principles or phenomena.

2.3.4 Procedure for pedagogical analysis: The cycle of the pedagogical analysis as follows:



2.3.4.1 Classification of content

Every text book of Mathematics deals contents in units. The complete syllabus is divided into various units. Every unit deals with facts, concepts, rules, formulae etc.

Subject matter or contents involve major and minor concepts. All the concepts are properly sequenced so that relevant teaching-learning activities may be designed to achieve specific instructional and objectives. Analysis of concepts into major and minor concepts is essential. Analyse the unit in to major and minor concepts keeping in view following questions:

- Are the contents essential for students?
- Are the content relevant?

- Are the content written in a useful sequence?

Take in to consideration the following points while selecting and sequencing of major and minor concepts:

- From simple to complex.
- From concrete to abstract.
- From practical experiences to theory.
- From examples to formulae.
- Child centeredness.

Example: 1 Content analysis of Central Tendencies

Identification of Major and Minor Concepts

1 Major Concept : Concept of central tendencies

Minor Concepts:

- Meaning of the term central tendencies.
- Need of central tendencies.
- Uses of central tendencies.
- Organize the given data in suitable class interval.

2 Major Concept: Mean

Minor Concepts:

- Meaning of the term mean.
- Differentiate between grouped data and ungrouped data.
- Finding out value of mean for grouped and ungrouped data.
- Finding out any term if its mean is given.
- Uses of mean.
- Relation between mean and average.

3 Major Concept: Median

Minor Concepts:

- Meaning of the term median.
- Difference between grouped and ungrouped data.
- Finding out the value of median for grouped and ungrouped data.
- If median is given then finding out any missing term.
- Uses of median.
- Relation between mean and median.

4 Major Concept: Mode

Minor Concepts:

- Meaning of the term mode.
- Finding out the value of mode for grouped and ungrouped data
- If mode is given then finding any missing term.
- Uses of mode.
- Relation between mean, median and mode.

2.3.4.2 Objectives Formulation

After Analysis of concepts of a unit in to major and minor concepts, the next step is objective formulation in the light of teaching of identified concepts. The activities, methods, techniques, role of the teacher and the learners, measuring tools, etc. are decided after objective formulation in the light of the identified major and minor concepts with the help of content analysis. Objectives are specific, short term results seen as intermediate steps in reaching the goals. They are practical, concrete and specific. Objectives must be expressed in clear, precise language and they must be measurable.

Bloom (1956) Explicit formulations of ways in which students are expected to be changed by the educative process.

Gallagher and Smith (1989) properly constructed education objectives represent relatively specific statements about what students are able to do following instruction.

Characteristics of objectives are listed below (Westberg and Jason, 1993)

1. Objectives are clearly stated.

2. Objectives are consistent with overall goals of the school
3. Objectives are realistic.
4. Objectives are selected in line with the stages of development.
5. Objectives should be comprehensive.

Objectives are the expected learning outcomes after teaching: learning Objectives, educational Objectives, curriculum Objectives, performance Objectives, operational Objectives, Instructional Objectives, etc. are the other terms in use.

Example: After going through the contents of the topic “central tendencies” students are expected to demonstrate the following types of behavioral outcomes.

- recall the concept central tendencies.
- list the uses of the central tendencies.
- differentiate between grouped and ungrouped data.
- recall the terms like class interval, frequency, cumulative frequency, median group, modal group etc.
- recall the formula for finding mean, median and mode by short method as well as long method.
- differentiate between mean, median and mode.
- find out any missing terms if mean is given.
- find out any missing term if median or mode is given.
- demonstrate their skill of utilizing the concept.
- Compute mean, median and mode in solving their day to day life problems.

2.3.4.3 Choosing Methods, Materials and Learning experience

What is the best method to teach a certain topic? Or How can I enable children to learn Mathematics? These are some of the questions for which every teacher wants to find a solution. Different methods of teaching mathematics have been proposed by different educators and the knowledge of these methods may help in working out a better teaching strategy. It is not appropriate for a teacher to commit to one particular method. A teacher should adopt a teaching approach

after considering the nature of the children, their interests and maturity and the resources available. Every method has certain merits and few demerits and it is the work of a teacher to decide which method is best for the students. Some of the methods of teaching Mathematics are as follows:

- Lecture Method
- Inductive-Deductive Method
- Heuristic Method (Discovery/Inquiry Method)
- Analytical-Synthetic Method
- Project Method
- Brain Storming
- Think-Pair-Share
- Play way method
- Learning by Doing
- Problem Solving Approach
- Laboratory Method
- Experimental approach
- Constructive approach, strategies and materials.

All the above mentioned methods may not be equally appropriate and suitable for all levels of mathematics teaching. The teacher, after knowing about all these methods, their merits and demerits, should be able to make his/her own method by imbibing the good qualities of all the methods.

The method finally adopted by the teacher must ensure maximum participation of the child, proceed from concrete to abstraction and provide knowledge at the understanding level.

Materials include textbooks, video and audio tapes, computer software, and visual aids. They influence the content and the procedures of learning. The choice of deductive vs. inductive learning, the role of memorization, the use of creativity and problem solving, production vs. reception, and the order in which materials are presented are all influenced by the materials

Though students should be the center of instruction, in many cases, teachers and students rely on materials, and the materials become the center of

instruction. Since many teachers are busy and do not have the time or inclination to prepare extra materials, textbooks and other commercially produced materials are very important in language instruction. Therefore, it is important for teachers to know how to choose the best material for instruction, how to make supplementary materials for the class, and how to adapt materials.

Learning experience, method and material in mathematics should be selected with due consideration to situations which will

- i. Fit students for useful service to the community.
 - ii. Discover vocational possibilities.
 - iii. Develop the ability to analyze and to solve the problems of life.
- **Example:** Inductive method, deductive method, lectures cum demonstration method, laboratory method, project method etc.
 - Explanation narration and demonstration devices, experimentation and surveying techniques.
 - **Materials and learning experience** To start with interesting and useful examples from the day to day life of the students will be used e.g school attendance board, merit board, test scores of a match played between two teams, the chart or table showing pass percentage of boys and girls in a public examination.
 - Students will be provided the data in the form of statistical tables, frequency distribution and graphical forms.
 - Problem will be solved with the help of charts, pictures, transparencies models etc.
 - Students will be asked to analyze the nature of data like achievement scores of the students of a class, the measure of height, weight etc. of a group of cricket players, by properly organizing and classifying them in a serial order. They will discover that there are few individuals who either score very high or very low. The marks and scores of most of the individuals lie somewhere at the centre i.e. midway between the highest and the lowest scores of the whole group .The tendency of the group is known as central tendency.

2.3.4.4 Evaluation Procedures, Tools and Devices

Evaluation is the last step in Pedagogical Analysis. Whenever we do some work, we always set objectives for that work. For fulfillment of these objectives we start our work in a planned manner for a specific time period. After doing that work for some time, we become curious to know that how much success we have got and to what extent we are able to achieve our objective. In this way we check our work to know whether we are using our energy and ability in right direction or not? This we call as evaluation.

Evaluation is a process by which we come to know that to what extent the objectives are achieved by classroom teaching. Measurement is included in evaluation. Evaluation measures children's whole personality and behavior. The process of evaluation completes only when the suitable decision can be taken for which the objectives were set.

Evaluation helps you to:

- name the different techniques and sources of evaluation and helps to select the appropriate technique of evaluation according to the need.
- select and use of general and specific teaching- learning method for the teaching of specific topics.
- evaluate both cognitive and non-cognitive aspect of child's personality.
- analyze the results of continuous and comprehensive evaluation and through remedial teaching, be able to improve the achievement and ability level of the children.
- report progress of the learner to parents, students, counselors, principal for taking appropriate actions to improve the performance of the learner.

To assess the progress of students towards the accepted goal, a variety of evaluation techniques are applied, for example:

1 Oral Test :

These tests are based on oral communication between the teacher and student. Here in general oral questions are put to the students for being responded by them in oral form.

2 Practical Tests:

In these tests students are required to demonstrate their learning performance by engaging themselves in experimental and work activities.

3 Written Test:

These tests require the use of writing material like paper and pen, Students receive test questions through written media known as question paper and are required to give their responses on the supplied answer sheet or the question paper itself in the written form. The scoring and interpretation of these answer sheets or written responses thus becomes a basis for the evaluation to the teaching learning outcomes and abilities of the students.

Example: The following items will be used for the necessary evaluation work

1. What do you mean by central tendencies?
2. What are the types of central tendencies?
3. Mention the relevant formula for finding
 - (i) Mean (ii) Median (iii) Mode
 for grouped data and ungrouped data.
4. The number of children in 10 families of a locality are
2, 4, 3, 4, 2, 0, 3, 5, 1, 6
Find the mean number of children per family.
5. The average of 7, 9, 0, x, 6 is 5.4 Find x.
6. Find the median of 2, 10, 9, 9, 5, 2, 3, 7, 11.
7. Find the median from the following data
19, 25, 59, 48, 35, 31, 30, 32, 51
If 25 is replaced by 52, what will be the new median?
8. What is the formula for mode when mean and median are given?
9. What are the uses of mean, median and mode?
10. Find mean, median and mode of the following frequency distribution

x:	3	6	10	12	7	15
f:	3	4	2	8	13	10

[NOTE: For comprehensive evaluation prescribed books may be consulted according to their class]

2.3.5 Summary

This unit discusses the components and operation involved in the task of pedagogical analysis. The analysis of a given content material in the subject mathematics carried out well in the spirit of the science of teaching is known by the term pedagogical analysis of the content in Mathematics.

Pedagogical analysis of a unit in Mathematics involves the following five components:

- 1 Content analysis of the unit to identify the major and minor concepts.
- 2 Writing the instructional objectives using behavioral terms.
- 3 Designing of learning experiences.
- 4 Selection of appropriate methods, techniques, activities and materials to achieve the instructional objectives.
- 5 Selection of evaluation procedures, tools and devices in order to evaluate the achievement of instructional objectives.

All the five steps of pedagogical analysis are strongly inter-related.

2.3.6 Suggested Questions

1. What do you mean by pedagogical analysis of a content material in the subject Mathematics?
2. Explain clearly the difference between content analysis and pedagogical analysis.
3. What do you mean by the term “pedagogy”?
4. What are the main four components involved in the process of pedagogical analysis of a given content material in the subject?

2.3.7 Suggested Readings and Web Resources

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Lesson No. 2.4

Pedagogical Analysis of Certain Unit in Mathematics - I**Structure of the Lesson**

2.4.1 Introduction

2.4.2 Objectives

2.4.3 Presentation of contents

2.4.3.1 Pedagogical Analysis: Arithmetic Ratio and Proportion

2.4.3.2 Identification of Major and Minor Concepts

2.4.3.3 Objective Formulation

2.4.3.4 Choosing Methods, Materials and Learning Experiences

2.4.3.5 Evaluation Procedure

2.4.3.2 Pedagogical Analysis :Algebra Linear and Quadratic Equation

2.4.3.1 Identification of Major and Minor Concepts

2.4.3.2 Objective Formulation

2.4.3.3 Choosing Methods, Materials and Learning Experiences

2.4.3.4 Evaluation Procedure

2.4.4 **Suggested Questions**2.4.5 **Suggested Readings****2.4.1 Introduction**

You have discussed in lesson-9 about basic components of pedagogical analysis of a unit in Mathematics. The unit may be divided to major and minor concepts involved in it. The learning experiences methods, techniques and material recourses for teaching minor and major. Contents can be decided after determining the behavioral objectives for these minor and major concepts. Finally, you must also take care of evaluation aspects of these minor and major concepts both at the beginning and the closing of the lessons on minor and major concepts. Evaluation procedures, tools and devices must be decided. The pedagogical analysis of the unit on this pattern help you in making your learning effective and learners may be helped to acquire the concepts in line with stipulated instructional objectives. The present lesson deals with a few

examples of units linked with 8th and 9th class for further illustration of the concept of pedagogical analysis. Units taken as examples in this lesson are: Arithmetic (ratio and proportion), Algebra (linear and quadratic equation).

2.4.2 Objectives

After reading this lesson you will be able to:

- explain main four components involved in the process of pedagogical analysis of a given content material in the unit.
- analyze the contents of the unit into concepts.
- select-learning experiences, methods and materials for learning the relevant concepts.
- select –appropriate evaluation procedure tools and devices to measure the learning outcomes.
- analyse pedagogically certain units e.g linear and quadratic equations and ratio and proportion.

2.4.3 Presentation of contents

2.4.3.1 Pedagogical analysis: Arithmetic : ratio and proportion

2.3.3.1.1 Identification of Major and Minor Concepts

1. Major Concept: ratio

Minor Concepts:

- Meaning of the term ratio.
- Need of ratio.
- Difference between ratio and fraction.
- Defining the term antecedent and consequent

2. Major Concept: Composition of ratio

Minor Concepts:

- Compounded ratio.
- Duplicate ratio.
- Sub duplicate ratio.

- Sub triplicate ratio.
 - Reciprocal ratio.
3. Major Concept: Comparison of ratio
- Minor Concepts:
- Ratio of greater inequality.
 - Ratio of less inequality.
 - Ratio of equality.
4. Major Concept: Finding ratio between two quantities
- Minor Concepts:
- Converting the quantities into similar units.
 - Simplifying the result.
 - Solve problem of ratio related to day by day life.
5. Major Concept: Proportion
- Minor Concepts:
- Meaning of proportion.
 - Need of proportion.
 - Difference between ratio and proportion.
 - Define the term extremes and means.
6. Major Concept: Types of proportion
- Minor Concepts:
- Direct proportion.
 - Inverse proportion.
 - Difference between Direct proportion and Inverse proportion.
7. Major Concept: Some results on proportion
- Minor Concepts:

- Inverendo.
- Alternendo.
- Comonendo
- Dividendo.
- Componendo and dividend

2.4.3.1.2 Objective Formulation

After going through the concept of the topic “Ratio and Proportion” students are expected to demonstrate the following types of behavioral outcomes.

- i. recall the concept of “ratio”.
- ii. tell need of ratio.
- iii. differentiate between ratio and fraction.
- iv. explain composition of ratio.
- v. differentiate compounded ratio, sub triplicate ratio, reciprocal ratio.
- vi. compute ratio of greater inequality, less inequality and ratio of equality.
- vii. demonstrate skill of calculating ratio between two quantities.
- viii. demonstrate ability to solve problems related to ratio in daily life.
- ix. recall the concept of “proportion”.
- x. tell the need of proportion.
- xi. differentiate between ratio and proportion.
- xii. explain the term extremes and means.
- xiii. recall the relationship between extremes and means.
- xiv. explain the meaning of direct proportion.
- xv. explain the meaning of inverse proportion.
- xvi. differentiate between direct proportion and inverse proportion.

- xvii. recall some results of proportion.
- xviii. demonstrate the ability to solve problems related to proportion in daily life.

2.4.3.1.3 Choosing Methods, Materials and Learning Experiences

1. Methods

- Inductive method, analytical method, research method, lecture cum – demonstration method, problem solving method
- Explanation and narration devices.

2. Learning Experiences and Materials

- Teaching –learning of the topic ratio can be more interesting and useful to the students by discovering its links with daily uses as well as with the teaching- learning experiences related to other areas of school, curriculum.
- The concept of ratio is commonly used in making different types of recipes. In these ingredients are always mixed by following the ratio pattern.
- Difference between ratio and fraction will be made clear through relevant examples with the help of blackboard, transparency and presentation.
- Ratio between two quantities can be computed by dividing one with another and then simplifying the obtained result. This fact may be deduced by providing experiences to solve the practical and useful problems.
- Proportion can be explained through equivalent ratios that are responsible for giving birth to the concept of proportion.
- Students must also be told that the proportion existing between the four quantities a,b,c,d may also be written as $a:b::c:d$ conveying the same sense $a:b=c:d$ i.e the ratio between a and b is equal to the ratio between c and d.
- Difference between direct proportion and inverse proportion will be made clear through relevant examples with the help of black boards and transparency presentation etc.
- Describe the problem to be solved with the help of charts, pictures, transparencies, models etc.

2.4.3.1.4 Evaluation Procedure:

All the three oral, practical and written (essay type, short answer type, objective type) modes of testing and evaluation will be used for evaluating the students experiences regarding teaching learning of the topic “Ratio and Proportion”. Evaluation will be done in the beginning and the closure of the lessons/units.

- 1 What do you mean by ratio?
- 2 What is the difference between ratio and fraction ?
- 1 What do you understand by antecedent and consequent ?
- 2 Divide Rs. 162 in the ratio 2:7.
- 3 A bag contains Rs 187 in the form of 1 rupee, 50 p and 10 p coins in the ratio 3:4:5. Find the number of each type of coins.
- 4 The ratio between A and B’s shares is 7:11. If B’s share is Rs 22055. Find A’s share.
- 5 What do you mean by proportion?
- 6 If $x:6::5:3$, find the value of x.
- 7 What is the difference between ratio and proportion?
- 8 Find two numbers such that the mean proportion between them is 28 and the third proportion to them is 224.
- 9 In a mixture of 45 liters, the ratio of milk and water is 4:1. How much water must be added to make the ratio 3:2 ?
- 10 Match the duplicate ratio

1. 1:4	2:3
2. 3:4	1:16
3 $\sqrt{2}: \sqrt{3}$	81:256
4. 9:16	9:16

2.4.3.2 Pedagogical analysis: Algebra : linear and quadratic equation**2.4.3.2.1 Identification of Major and Minor Concepts****1. Major Concept: Meaning of the term “Equation”.**

Minor Concepts:

- Importance of solving equations.
- Types of equations.

2.3 Major Concept: Linear equation in one variable and two variable

Minor Concepts:

- What are linear equation in one variable and two variables?
- Identification of equation in one variable and two variables.
- Solution of a linear equation in one variable and in two variables.
- Graph of the equation in two variables equation.

3. Major Concept: Simultaneous linear equation

Minor Concepts:

- What do you understand by simultaneous linear equation?
- Consistent and inconsistent system of simultaneous linear equation.
- Solving simultaneous linear equation by graphical method.
- Solving simultaneous linear equation by (Algebraic method) like substitution method, elimination method .
- General solution and condition for solvability.
- Solve word problems on simultaneous linear equation.

4. Major Concept: Quadratic equations

Minor Concepts:

- Identification of quadratic equation.
- Roots of a quadratic equation.

- Solving a quadratic equation by factorization method.
- Solving a quadratic equation by completing the square quadratic formula (Shreedharacharyas rule).
- Nature of the roots of a quadratic equation.
- Equation reducible to quadratic forms.
- Solve word problems on quadratic equation.

2.4.3.2 Objective Formulation:

After going through the contents of the topic “Linear and quadratic equation” students are expected to demonstrate the following types of behavioral outcomes.

- i. recall and explain the concept equations.
- ii. tell the need and importance of solving the linear equation and quadratic equation.
- iii. tell about the various types of equations.
- iv. tell the linear equation in one and two variables.
- v. identify the equation in one variable and two variables.
- vi. tell about the simultaneous linear equation.
- vii. give examples to explain consistent and inconsistent system of simultaneous linear equation.
- viii. tell different method of solving simultaneous linear equation.
- ix. solve problem on linear equation.
- x. define the quadratic equations.
- xi. identify the quadratic equation.
- xii. recall the formula for finding roots of the quadratic equation.
- xiii. tell different method of solving quadratic equation.
- xiv. solve word problem related to quadratic equation.
- xv. transform the equation to quadratic forms.

2.4.3.2.3 Choosing Methods, Materials and Learning Experiences

1 Methods

- Inductive-deductive method, Heuristic method, laboratory method, lecture cum demonstration method.
- Explanation, narration and demonstration devices.

2 Learning Experiences and Materials:

- Every equation has two sides like a balance. The help of a balance may be taken for illustrating many essential facts about equations.
- For word problems students must be asked to study the problem carefully and to analyze what is given and to be find out.
- For explaining simultaneous equations a few interesting and practical problem like “there are some birds on two trees- one mango tree and other neem tree. The birds sitting on mango tree said to be birds of neem tree “If five of you come to us our number will become double than you, “ The birds on neem tree replied, “if five of you come to us we will become equal to you,” By the help of this example we find how many birds were sitting on each tree.
- Demonstrate the procedure of solving linear, simultaneous and quadratic equation with the help of blackboard, graphic and transparency presentation.
- Describe the problem to be solved with the help of charts, pictures, transparencies models etc.
- Students are familiar with the methods of solving equation i.e simultaneous linear equation and quadratic equation related to day to day life.

2.4.3.4.4 Evaluation Procedure: All the three oral, practical and written (essay type, short answer type, objective type) modes of testing and evaluation will be used for evaluating the students experience regarding teaching learning of the topic “Linear and quadratic equation”. Evaluation will be done both in the beginning and closing of the lesson/unit.

1. What do you mean by equation?
2. What are the types of equation? Explain them.

3. Draw the graph of the equation $5x-3y=1$.

4. Solve the equation

i) $x+y=7$

ii) $x-y= 2$

$3x-2y=1$

$2x-y =6$

5 Find the value of K, if it exists such that the systems of equation

$Kx+3y=K-3$, $12x+Ky=K$ has infinitely many solutions

1 Solve the equation

(i) $x^2-x+1=0$

(ii) $x^2-1/x^2=5$

2 Determine K so that the equation $x^2-4x+K=0$ has distinct real roots.

3 If α , β are the roots of equation $2x^2-3x+1=0$, forms an equation whose roots are α/β and β/α .

4 Solve $\sqrt{25-x^2} = x-1$.

5 The sum of two numbers is 129 and their difference is 35. Find the numbers.

6 One year ago, a man was 8 times as old as his son. Now his age equal to the square of his son's age. Find their present age.

7 Fill in the blanks

(i) A pair of linear equation in two variables is said to form a system of equation.

(ii) The sum of two numbers is 129 and their difference is 30. The number

(iii) $P(x)= Ax^2+bx+c$ be a quadratic polynomial and let α be a real number such that $p(\alpha)=0$ then α is called the of the polynomial $p(x)$.

(iv) For the quadratic equation $ax^2+bx+c=0$, $a \neq 0$ the expression..... is called the discriminant.

[NOTE : For comprehensive evaluation prescribed books may be consulted according to their class]

2.4.4 Suggested Questions

1. How will you carry out the pedagogical analysis of the content material related to the unit “linear equation”?
2. Describe the procedure of pedagogical analysis of the unit “Quadratic equation”
3. How will you carry out the pedagogical analysis of the content material related with to the unit “Ratio and proportion”?

2.5 Suggested Readings.

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Lesson No. 2.5

Pedagogical Analysis of Certain Unit in Mathematics - II

Structure of the Lesson

- 2.5.1 Introduction
- 2.5.2 Objectives
- 2.5.3 Presentation of contents
 - 2.5.3.1 Pedagogical Analysis : Geometry (Congruency)
 - 2.5.3.1.1 Identification of Major and Minor Concepts
 - 2.5.3.1.2 Objective Formulation
 - 2.5.3.1.3 Choosing Methods, Materials and Learning Experiences
 - 2.5.3.1.4 Evaluation Procedure
 - 2.5.3.2 Pedagogical Analysis : Trigonometry
 - 2.5.3.2.1 Identification of Major and Minor Concepts
 - 2.5.3.2.2 Objective Formulation
 - 2.5.3.2.3 Choosing Methods, Materials and Learning Experiences
 - 2.5.3.2.4 Evaluation Procedure
 - 2.5.3.3 Pedagogical Analysis : Statistics (Central Tendencies)
 - 2.5.3.3.1 Identification of Major and Minor Concepts
 - 2.5.3.3.2 Objective Formulations
 - 2.5.3.3.3 Choosing Methods, Materials and Learning Experiences
 - 2.5.3.3.4 Evaluation Procedure
- 2.5.4 Summary
- 2.5.5 **Suggested Questions**
- 2.5.6 **Suggested Readings**

2.5.1 Introduction

You have discussed in lesson-9 about basic components of pedagogical analysis of a unit in Mathematics. The unit may be divided to major and minor concepts involved in it. The learning experiences methods, techniques and material

recourses for teaching minor and major. Contents can be decided after determining the behavioral objectives for these minor and major concepts. Finally, you must also take care of evaluation aspects of these minor and major concepts both at the beginning and the closing of the lessons on minor and major concepts. Evaluation procedures, tools and devices must be decided. The pedagogical analysis of the unit on this pattern help you in making your learning effective and learners may be helped to acquire the concepts in line with stipulated instructional objectives. The present lesson deals with a few examples of units linked with 8th and 9th class for further illustration of the concept of pedagogical analysis. Units taken as examples in this lesson are: Geometry, Trigonometry (Congruency), Statistics (Central tendencies).

2.5.2 Objectives

After reading this lesson you will be able to:

- explain main four components involved in the process of pedagogical analysis of a given content material in the unit.
- analyze the contents of the unit into concepts.
- select-learning experiences, methods and materials for learning the relevant concepts.
- select –appropriate evaluation procedure tools and devices to measure the learning outcomes.
- analyse pedagogically certain units e.g central tendencies, congruency, trigonometry.

2.5.3 Presentation of contents

2.5.3.1 Pedagogical Analysis: Congruency

2.5.3.1.1 Identification of Major and Minor Concepts:

1. Major Concept: Significance of congruency

Minor Concepts:

- Recognize congruent figures.
- Symbol of congruent figures.
- Size and shape of congruent figures.

2. Major Concept: Area of congruent figures

Minor Concepts:

- Area of congruent figures are equal.
- Area of non- congruent figures may be equal.

3. Major Concept: Congruency of geometrical figures

Minor Concepts:

- Congruency of two line segment.
- Congruency of two angles.
- Congruency of quadrilateral like square, rectangle, parallelogram, etc.
- Congruency of circle.

4. Major Concept: Congruency of Triangles

Minor Concepts:

- Sufficient condition (criteria) for congruence of two triangles.
- SAS (Side –Angle-Side) congruency.
- ASA (Angle-Side-Angle) congruency.
- AAS (Angle-Angle-Side) congruency.
- SSS (Side-Side-Side) congruency.
- RHS (Right-Angle-Hypotenuse-Side) congruency.

2.5.3.1.2 **Objective Formulation**

After studying the contents of the topic “congruency” students are expected to demonstrate the following types of behavioral outcomes.

- state when two figures are congruent.
- give examples of the congruent figures from the surrounding.
- explain the concept of super imposition.
- recall the symbol of congruency.
- demonstrate clearly by giving proof or examples that any two congruent figures have equal area.

- demonstrate that area of non-congruent figures may be equal.
- explain the congruency of different geometrical figures like line segment, angles, quadrilateral, circle etc.
- recite the rules of congruency of two triangles
- explain all the conditions for proving the congruency of two triangles.
- demonstrate the use of congruency of the triangles for proving theorems in geometry
- identify the condition of congruency in the given problems.

2.5.3.1.3 Choosing Methods, Materials and Learning Experiences

1. Methods

- Demonstration-Cum-Discussion method, laboratory method.
- Explanation and demonstration devices, observation technique.

2. Learning Experiences and Materials

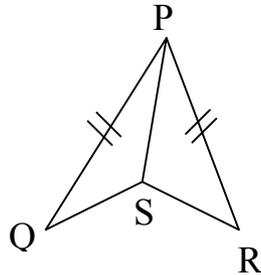
- Students will be introduced with the concept of congruency. The beginning can be made with the demonstration of so many objects available in their environment having an identical shape and size like racket, coin, etc.
- Examples the concept of super imposition will be given. Students may well realize after experiment that congruent figures have equal areas. However the converse is not true that of two geometrical figures have equal area that may or may not be congruent.
- Geometrical figures congruent by making congruent shapes of card, chart paper and card-board, etc will be used to prove geometrical figures.
- Show clearly all the condition of congruency by giving proof and examples.

2.5.3.1.4 Evaluation Procedure

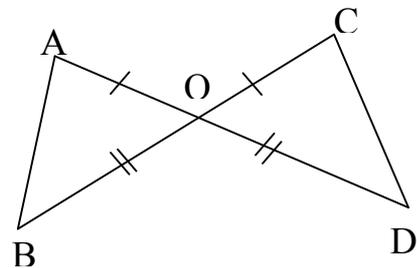
All the three oral, practical and written (essay type, short answer type, objective type) modes of testing and evaluation will be used for evaluating the teaching-learning outcomes of the contents related to the topic “congruency” both in the beginning and closing of the lessons/ unit. The following items will be used for the necessary evaluation work

1. What do you mean by congruent figures?
2. What is the symbol of congruency?
3. Give examples of congruent figures from day to day life.
4. What do you mean by congruency of two triangles?
5. In the figures below the congruent parts of triangles have been indicated by line markings. State by which congruence criterion, the triangles are congruent

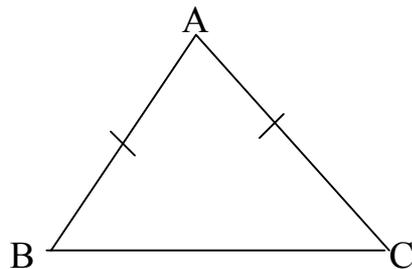
(a)



(b)



6. In $\triangle ABC$, let $\angle A = 120^\circ$ and $AB = AC$ find $\angle B$ and $\angle C$



7. Prove that angles opposite to two equal sides of a triangle are equal.
8. In $\triangle ABC$, $AB = AC$ and the bisector of angles B and C intersect at the point O. Prove that $BO = CO$ and ray AO is the bisector of angle BAC.
9. ABCD is a parallelogram. If the two diagonals are equal, find the measure of angle ABC.
10. For each of the following state whether the statement is true or false
 - i. If two angles of a triangle are equal then the sides opposite to them are also equal.

- ii. If two sides and one angle of a triangle are equal to two sides and one angle of another triangle, then the two triangles are congruent.
- iii. If the sum of two sides of one triangle is equal to the sum of two sides of another triangle, then their third sides must be equal.
- iv. If the area of two triangles are equal, the triangles may not be congruent.
- v. If area of two triangles are equal and their corresponding angles are also equal then the triangles are congruent.

11 Fill in the blanks

- i. In a triangle ,theangle has longer side opposite to it.
- ii. In a triangle, the side has greater angle opposite to it.

[NOTE : For comprehensive evaluation prescribed books may be consulted according to their class]

2.5.3.3 Pedagogical Analysis: Trigonometry

2.5.3.1.1 Identification of Major and Minor Concepts

1. Major Concept: Significance of trigonometry

Minor Concepts:

- Meaning of the term trigonometry.
- Scope of trigonometry.
- Meaning of trigonometry function.
- Use of trigonometric function.
- What do you mean by Pythagoras theorem?

2. Major Concept: Trigonometry Ratios

Minor Concepts:

- Definition of trigonometry ratios.
- Trigonometry ratios of complement angles.
- Trigonometry ratios for standard values.

- Standard value table.
- Express all trigonometry ratios in terms of one of them.

3. Major Concept: Trigonometric Identities

Minor Concepts:

- Meaning of trigonometric identities.
- Standard identities.
- Use of basic identities.
- Methods of proving other identities.

4. Major Concept: Height and Distance

Minor Concepts:

- Meaning of terms angle of elevation.
- Meaning of terms angle of depression.
- Application- drawing a correct and proper diagram for the given problem.
- Solution of triangles.

2.5.3.2.1 Objective Formulation: After analyze the contents of the topic “Trigonometry” next step is objective formulation. After going through the contents students are expected to demonstrate the following types of behavioral outcomes.

- i. recall and explain the concept trigonometry.
- ii. recall the major points linked with the scope of trigonometry.
- iii. explain the meaning of trigonometry functions.
- iv. solve the problems using Pythagoras theorem involving trigonometry ratio.
- v. demonstrate the students how the concept of similarity from the basis of trigonometric ratios.
- vi. manipulate trigonometric ratios and appreciate their relationship.
- vii. find values of trigonometric ratio for standard value of angles.

- viii. design the standard value table.
- ix. prove trigonometric identities,
- x. derive basic identities.
- xi. solve problems on height and distance.

2.5.3.2.1 Choosing Methods, Materials and Learning Experiences

1. Methods

- Discussing with inductive reasoning, deductive method, lecture method, laboratory method, project method etc.
- Explaining narration and demonstration devices, experimentation and surveying techniques.

2. Learning Experiences and Material

- In the beginning the function of an acute angle may be defined in term of the sides of a right angle containing that angle.
- Students will be asked to measure the angles and sides of right triangles and compute the numerical values of the ratio representing the sine, cosine, tangent, cotangent, secant and cosecant.
- Students will be asked to draw correct diagram and using it to find height and distances.
- Puzzles of trigonometric tables will be used to teach standard value table.
- Student will be helped to prepare trigonometric ratio table for standard angle $0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° .
- Information will be given about three basic trigonometric identities which are derived from the Pythagoras theorem.
- Asked to solve the problems with the help of charts, pictures, transparencies models etc.

2.5.3.2.4 Evaluation Procedure- Evaluation procedure will be used both before and after teaching

- (1) What do you mean by trigonometry?

- (2) What is the relation between $\sin\theta$, $\cos\theta$, and $\tan\theta$?
- (3) If $\sin A = 2/3$, find the other two ratios, i.e $\cos A$ and $\tan A$.
- (4) If $\tan A = 1$ and $\tan B = \sqrt{3}$, evaluate $\cos A \cos B - \sin A \sin B$.
- (5) If $\sin\theta = 3/5$, find the value of $\tan^2\theta + \sin\theta \cos\theta + \cot\theta$.
- (6) Evaluate the following expression
- $\operatorname{cosec}^2 30^\circ \sin^2 45^\circ - \sec^2 60^\circ$
 - $\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$
- (7) Verify each of the following
- $\sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ$
 - $\tan 60^\circ = 2 \tan 30^\circ / (1 - \tan^2 30^\circ)$
- (8) What do you mean by angle of elevation and angle of depression?
- (9) At a point 20m away from the foot of a tower, the angle of elevation of the top of the tower is 30° , Find the height of the tower.
- (10) A shadow of a tower is $\sqrt{3}$ times the height of the tower. Find the angle of elevation of the sun.
- (11) For each of the following state whether the statement is true or false
- $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ = \sin 90^\circ$
 - $\sin 90^\circ = 3 \sin 30^\circ - 4 \sin^3 30^\circ$
 - $\sin 2A = 2 \tan A / (1 + \tan^2 A)$
 - $(1 - \sin^2 A) \sec^2 A = 1$

[NOTE : For comprehensive evaluation prescribed books may be consulted according to their class]

2.5.3.3 Pedagogical Analysis ; Central Tendencies

2.5.3.3.1 Identification of Major and Minor Concepts

5 Major Concept : Concept of central tendencies

Minor Concepts:

- Meaning of the term central tendencies.
- Need of central tendencies.
- Uses of central tendencies.
- Organize the given data in suitable class interval.

6 Major Concept: Mean

Minor Concepts:

- Meaning of the term mean.
- Differentiate between grouped data and ungrouped data.
- Finding out value of mean for grouped and ungrouped data.
- Finding out any term if its mean is given.
- Uses of mean.
- Relation between mean and average.

7 Major Concept: Median

Minor Concepts:

- Meaning of the term median.
- Difference between grouped and ungrouped data.
- Finding out the value of median for grouped and ungrouped data.
- If median is given then finding out any missing term.
- Uses of median.
- Relation between mean and median.

8 Major Concept: Mode

Minor Concepts:

- Meaning of the term mode.
- Finding out the value of mode for grouped and ungrouped data
- If mode is given then finding any missing term.

- Uses of mode.
- Relation between mean, median and mode.

2.5.3.3.2 Objective Formulation: After going through the contents of the topic “central tendencies” students are expected to demonstrate the following types of behavioral outcomes.

- recall the concept central tendencies.
- list the uses of the central tendencies.
- differentiate between grouped and ungrouped data.
- recall the terms like class interval, frequency, cumulative frequency, median group, modal group etc.
- recall the formula for finding mean, median and mode by short method as well as long method.
- differentiate between mean, median and mode.
- find out any missing terms if mean is given.
- find out any missing term if median or mode is given.
- demonstrate their skill of utilizing the concept.
- Compute mean, median and mode in solving their day to day life problems.

2.5.3.3.3 Choosing Method, Materials and Learning Experiences

1. Methods

- Inductive method, deductive method, lecture cum-demonstration method, laboratory method ,project method etc.
- Explanation narration and demonstration devices, experimentation and surveying techniques.

2. Learning Experiences and Materials:

- To start with interesting and useful examples from the day to day life of the students will be used e.g school attendance board, merit board, test scores of a match played between two teams, the

chart or table showing pass percentage of boys and girls in a public examination.

- Students will be provided the data in the form of statistical tables, frequency distribution and graphical forms.
- Problem will be solved with the help of charts, pictures, transparencies models etc.
- Students will be asked to analyze the nature of data like achievement scores of the students of a class, the measure of height, weight etc. of a group of cricket players, by properly organizing and classifying them in a serial order. They will discover that there are few individuals who either score very high or very low. The marks and scores of most of the individuals lie somewhere at the centre i.e midway between the highest and the lowest scores of the whole group .The tendency of the group is known as central tendency.

2.5.3.3.4 Evaluation Procedure

All the three oral, practical and written (essay type, short answer type and objective type) modes of testing and evaluation will be used for evaluating the teaching learning outcomes of the content related to the topic “central tendencies” both in the beginning as well as closing of the lesson.

The following items will be used for the necessary evaluation work

11. What do you mean by central tendencies?

12. What are the types of central tendencies?

13. Mention the relevant formula for finding

(i) Mean (ii) Median (iii) Mode

For grouped data and ungrouped data.

14. The number of children in 10 families of a locality are

2, 4, 3, 4, 2, 0, 3, 5, 1, 6

Find the mean number of children per family.

15. The average of 7, 9, 0, x, 6 is 5.4 Find x.

16. Find the median of 2, 10, 9, 9, 5, 2, 3, 7, 11.

17. Find the median from the following data

19, 25, 59, 48, 35, 31, 30, 32, 51

If 25 is replaced by 52, what will be the new median?

18. What is the formula for mode when mean and median are given?

19. What are the uses of mean, median and mode?

20. Find mean, median and mode of the following frequency distribution

x: 3 6 10 12 7 15

f: 3 4 2 8 13 10

[NOTE : For comprehensive evaluation prescribed books may be consulted according to their class]

[NOTE : For comprehensive evaluation prescribed books may be consulted according to their class]

2.5.4 Summary

Pedagogical analysis of a few units from 9th class Mathematics have been taken as examples with the aim to develop understanding about the skill of pedagogical analysis of a unit.

Examples: Central tendencies

1. Identification of major/minor concepts
 - Concept of central tendencies
 - Mean as central tendency
 - Median
 - Mode
2. Objective formulation
3. Choosing methods, materials and learning experiences
4. Evaluation.

Similar steps have been followed in other examples i.e congruency, trigonometry; area; volume; linear and quadratic equation; ratio and proportion.

2.5.5 Suggested Questions

4. How will you carry out the pedagogical analysis of the content material related to the unit “central tendency”?
5. Describe the procedure of pedagogical analysis of the unit “congruent and similar triangles”
6. How will you carry out the pedagogical analysis of the content material related to the unit “t-ratio of trigonometry”?
7. How will you carry out the pedagogical analysis of the content material related with to the unit “Height and distance of trigonometry”?

2.5.5 Suggested Readings.

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